



## COMPARATIVE ANALYSIS OF LOCAL GEOID MODELLING IN HONG LOCAL GOVERNMENT AREA OF ADAMAWA STATE, NIGERIA

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### ABSTRACT

*This study evaluates the effectiveness of geometric and gravimetric approaches for determining geoid undulations and orthometric heights using integrated GPS/levelling data and selected Global Geopotential Models (GGMs). Three gravimetric techniques: Stokes' integral, the Simple Combine Method (SCM), and direct geoid extraction from GGMs were assessed using EGM2008, SGG-UGM-2, and XGM2019e\_2156 models to identify the most reliable method for accurate height determination. Statistical measures, specifically mean values and standard deviations, were employed to examine accuracy, consistency, and clustering of the derived geoid and height solutions relative to spirit levelling observations. The results show that the SCM consistently produced the lowest standard deviations across all evaluated GGMs, indicating superior precision in geoid modelling. This improved performance is attributed to the method's ability to effectively combine long-wavelength gravity information from GGMs with short-wavelength terrestrial gravity signals. Orthometric heights derived using the Stokes approach exhibited the closest agreement with spirit levelling, particularly when paired with the EGM2008 and SGG-UGM-2 models, demonstrating their suitability for regional height applications. Although XGM2019e\_2156 offers enhanced global resolution, observed systematic shifts in orthometric heights suggest the need for regional adjustment before local use. Overall, the findings confirm that integrated geometric gravimetric approaches, especially the SCM combined with regionally appropriate GGMs, significantly enhance the accuracy and reliability of geoid and orthometric height determination.*

**Keywords** Geoid modelling; Orthometric height; Stokes' integral; Spectral Combination Method; Global Geopotential Model; GPS/Levelling.

### 1.0

### INTRODUCTION

Accurate determination of heights relative to the Earth's gravity field is a fundamental requirement in geodesy, surveying, mapping, and geoscientific applications. Orthometric heights, which are referenced to the geoid as an equipotential surface of the Earth's gravity field, provide a physically meaningful representation of elevation and are indispensable for infrastructure development, flood risk assessment, hydrological modelling, and geospatial data integration (Lestari *et al.*, 2023; Ihde *et al.*, 2022). According to Kamorudeen *et al.* (2022),



orthometric heights have been obtained through precise spirit levelling combined with gravity observations. Although this method delivers high accuracy over short distances, it is labour-intensive, time consuming, and economically impractical for large scale or regional height determination (Tata & Ono, 2019). These limitations have motivated the adoption of satellite-based techniques, particularly the integration of Global Navigation Satellite System (GNSS) ellipsoidal heights with geoid models, as a more efficient alternative (Fajemirokun, 2023; Lestari et al., 2023; Tata & Ono, 2019).

Tata and Ono, (2018) reported that geometric approach to height determination relies on the relationship between ellipsoidal heights, orthometric heights, and geoid undulations. Studies by Janecka, (2023); Yoldoshaliyev, (2023) and Avsar *et al.* (2022) posited that through the combination of GNSS-derived ellipsoidal heights with independently measured orthometric heights, local or regional geometric geoids can be established. While this method is straightforward and effective in areas with dense levelling control, its reliability is highly dependent on the quality and spatial distribution of reference benchmarks, limiting its applicability in poorly surveyed regions (Janecka, 2023; Yoldoshaliyev, 2023). In contrast, gravimetric approaches derive geoid undulations from gravity field information and offer greater physical consistency over large areas. Among these, Stokes' integral has been widely applied for regional geoid modelling using terrestrial gravity anomalies corrected for topographic and atmospheric effects (Lemenkova, 2023). However, the classical Stokes formulation suffers from truncation and data coverage issues, particularly in regions with sparse gravity measurements (Lemenkova, 2023).

To overcome these challenges, modern techniques such as the Simple Combine Method (SCM) have been developed (Aleem, 2022). SCM optimally combines terrestrial gravity data with Global Geopotential Models (GGMs) in the spectral domain, thereby improving the representation of both long and short wavelength gravity signals (Aleem, 2022). This approach



has been shown to enhance geoid accuracy and stability, particularly in regional applications where data heterogeneity is common (Aleem, 2022). The development of high resolution GGMs from satellite gravimetry missions has significantly advanced global and regional geoid modelling. Models such as EGM2008 incorporate extensive terrestrial, airborne, and satellite gravity data, providing improved accuracy compared to earlier solutions (Janecka, 2023). More recent combined models, including XGM2019e\_2156, integrate satellite mission data with high resolution terrestrial observations, offering enhanced spatial resolution and spectral completeness (Yoldoshaliyev, 2023).

Despite these improvements, the performance of GGMs and gravimetric techniques varies across regions due to differences in data density, terrain characteristics, and gravity field complexity. Several studies have emphasized the necessity of local validation and comparative assessment of geoid modelling approaches to ensure their suitability for precise height determination (Aleem, 2022). In view of these, this study evaluates and compares geometric and gravimetric approaches for geoid and orthometric height determination using selected GGMs to identify the most reliable approach for accurate and consistent height determination in Hong LGA Adamawa State

## **2.0 MATERIALS AND METHODS**

### **2.1 Study Area Description**

#### **2.1.1 Location**

Hong Local Government Area (LGA) is located in the north-eastern part of Adamawa State, Nigeria, extending approximately between latitudes  $10^{\circ}00'N$  and  $10^{\circ}30'N$  and longitudes  $12^{\circ}45'E$  and  $13^{\circ}30'E$ . The LGA covers about 2,500 km<sup>2</sup> and is bounded by Gombi LGA to the west, Mubi to the east, Song to the south, and Askira/Uba LGA of Borno State to the north. Geographically and geodynamically, the area lies within the north-eastern segment of the



Benue Trench, where long-term tectonic activity and surface denudation have jointly influenced the present landforms and subsurface structure. This setting is particularly relevant for investigations involving gravity field behaviour and height system realization.

### **2.1.2 Topography**

The terrain of Hong LGA is characterized by gently to strongly undulating uplands dissected by narrow valleys, producing a landscape of moderate to high relief. Elevations across the upland surfaces generally range from about 700m to 1,100m, underlain predominantly by competent crystalline basement rocks. In contrast, the valley floors typically occur between 400m and 600m and consist mainly of weathered materials and recent alluvial deposits. Such pronounced elevation contrasts exert a strong influence on the spatial distribution of the gravity field. Elevated and rugged zones are commonly associated with relatively higher gravity responses due to the shallow presence of dense basement rocks, whereas low-lying areas tend to show reduced gravity values as a result of thicker regolith cover and lower average density.

These variations in gravity response provide a fundamental geophysical basis for spatial differences in geoid undulation across the study area. The marked relief contrast also enhances the separation of the geometric geoid signal, thereby offering favourable conditions for assessing the performance of GPS derived orthometric heights against geoid models obtained from global geopotential models (GGMs). Similar terrain controlled improvements in geoid detectability have been documented in other regions with complex topography.

### **2.1.3 Geology**

Hong LGA is underlain mainly by rocks of the Precambrian Basement Complex, forming part of the West African Craton. The dominant lithological units include gneisses, schists, quartzites, and granitic intrusions, with localized Tertiary to Quaternary sedimentary



deposits occupying valley bottoms. These basement rocks are extensively deformed, exhibiting foliation, folding, and faulting that reflect multiple phases of ancient tectonic activity.

The diversity of rock types and associated density contrasts contributes significantly to spatial variations in the gravity field. Consequently, the geological framework of Hong LGA provides an appropriate environment for examining how subsurface mass distribution influences geoid determination and orthometric height modelling when using both geometric and gravimetric approaches.

#### **2.1.4 Geodetic Relevance**

In the context of comparative geoid and orthometric height modelling, Hong LGA represents a highly suitable test area for evaluating the consistency between GPS derived heights and those obtained using global geopotential models such as EGM2008, SGG-UGM-2, and XGM2019e\_2156. The coexistence of elevated plains, isolated inselbergs, and low-lying valleys introduces significant variability in gravitational potential, allowing for a rigorous assessment of model sensitivity to terrain and density heterogeneity.

The area's topographic and geological diversity enables critical analysis of discrepancies between geometric and gravimetric solutions, thereby supporting a robust comparison of geoid models and their impact on orthometric height realization. Moreover, the availability of well-distributed GPS/levelling control points and relatively accessible terrain makes Hong LGA suitable for developing refined local or hybrid geoid models. Findings from such analyses are directly relevant to improving regional height systems and contribute to broader efforts aimed at aligning Nigeria's vertical reference framework with internationally accepted standards, including the International Height Reference Frame.

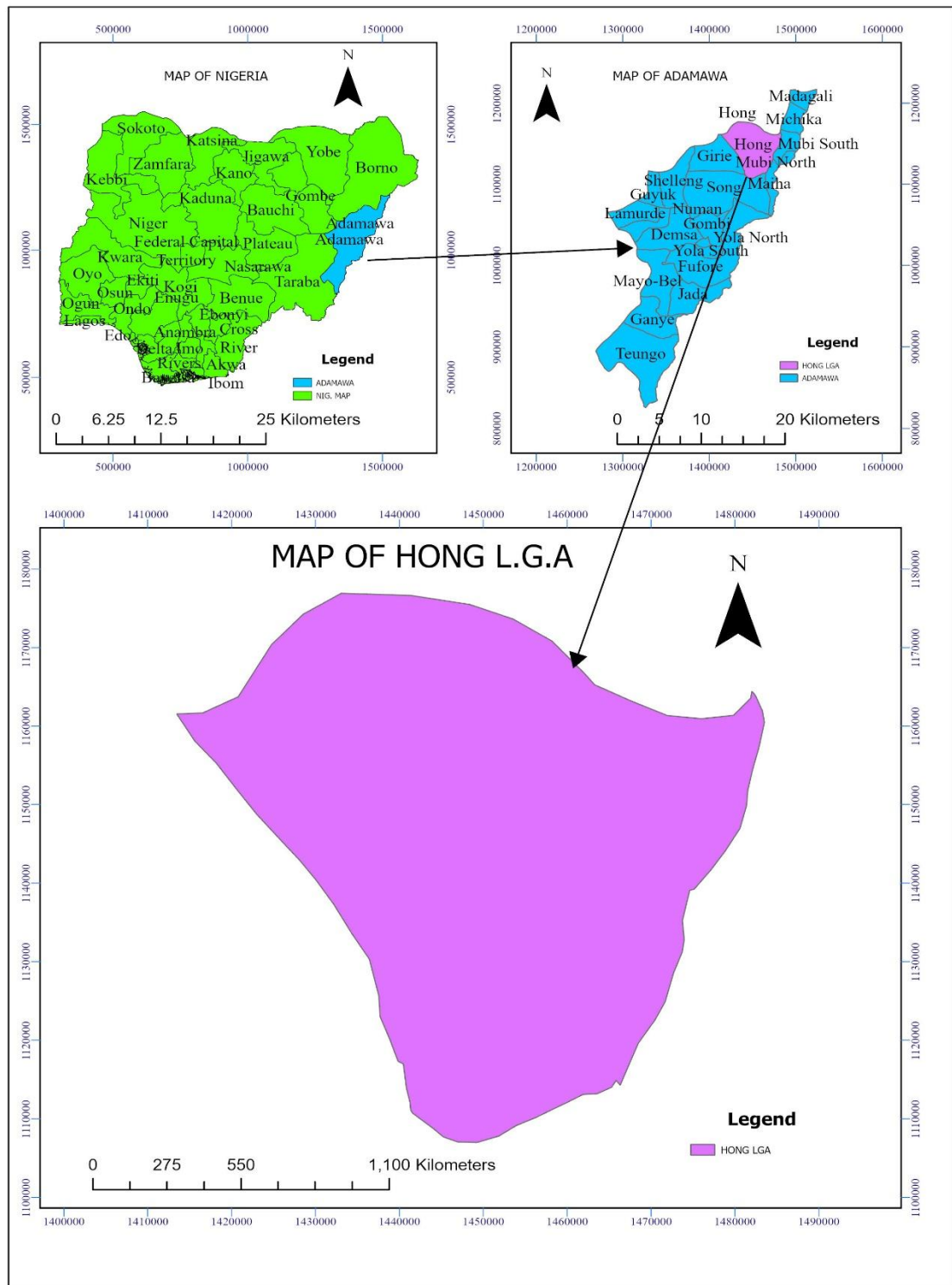


Figure 1 Map of the study area.



## **2.2 Experimental Procedure**

The study employed both geometric and gravimetric approaches for geoid and orthometric height determination. GPS observations provided ellipsoidal heights, while corresponding orthometric heights were obtained from precise spirit levelling at control points within the study area.

### **2.2.1 Geometric Approach**

The geometric geoid undulation was computed using the relation:  
$$N = h - H$$

Where:

$H$  is the orthometric height obtained from digital levelling,

$h$  is the ellipsoidal height obtained from GPS observation, and

$N$  is the geoid undulation the difference between the ellipsoid and the Orthometric height

### **2.2.2 Gravimetric Approaches**

Three gravimetric techniques were implemented:

- i. Stokes' Integral Method, which computes geoid undulations using terrestrial gravity anomalies and Stokes' kernel.
- ii. Simple Combine Method (SCM), which optimally blends terrestrial gravity data with GGMs to improve spectral completeness.
- iii. Direct GGM Approach, where geoid undulations were extracted directly from the selected GGMs

### 2.2.3 Global Geopotential Models

The gravimetric computations were performed using:

- i. EGM2008,
- ii. SGG-UGM-2, and
- iii. XGM2019e\_2156.

### 2.2.4 Statistical Analysis

Mean values and standard deviations of geoid undulations and orthometric heights were computed to assess accuracy, consistency, and clustering of results relative to spirit levelling observations.

## 3.0 RESULTS AND DISCUSSIONS

### 3.1 Results

Table 1 show the EGM2008 based geoidal undulation shows that, the mean values are 17.0514, 17.1131, 16.1193 and 17.131 for N\_Geometric, N\_Stokes, N\_SCM and N\_GGM respectively. Also, the standard deviations are 0.1631, 0.1648. 0.1481 and 0.1648 for N\_Geometric, N\_Stokes, N\_SCM and N\_GGM respectively. The N\_SCM with standard deviation of 0.1481 is more clustered to the mean and described as the best.

Table 1 Geoidal Undulation Statistics Based on EGM2008 Model

	<b>N_GEOMETRIC</b>	<b>N_STOKES</b>	<b>N_SCM</b>	<b>N_GGM</b>
<b>Mean</b>	17.0514	17.1131	16.1193	17.1131
<b>Std. Dev.</b>	0.1631	0.1648	0.1481	0.1648

Figure 1 shows the Orthometric height estimates from EGM2008, the mean values are 527.5496, 527.4880 and 528.4817 for H\_Spirit Levelling, H\_Stokes and H\_SCM respectively.

Also, the standard deviations are 2.2586, 2.2688 and 2.2676 for H\_Spirit Levelling, H\_Stokes and H\_SCM respectively. The Stokes method provides a nearly identical mean of 527.4880m. The Standard deviations across all methods remain similar, confirming uniformity in the estimates.

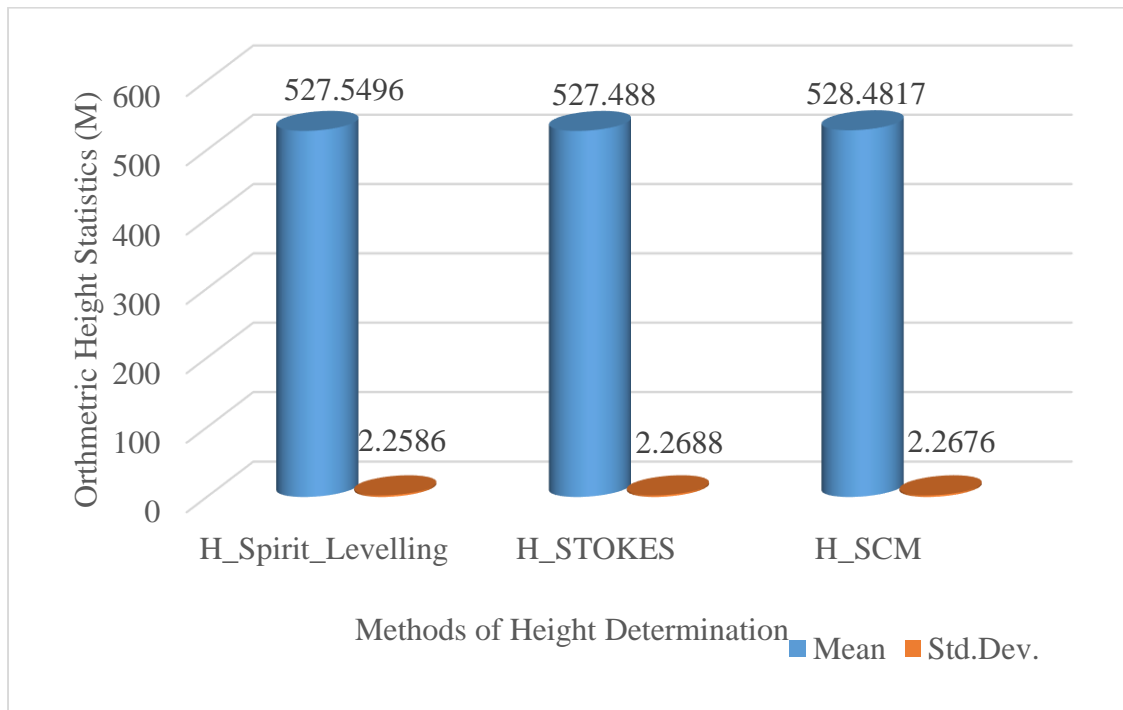


Figure 1. Orthometric Height Statistics Based on EGM2008 Model

Figure 2 shows the SGG-UGM-2 model based on geoidal undulation, the mean values are 17.0514, 17.0799, 16.0861 and 17.0799 for N\_Geometric, N\_Stokes, N-SCM and N GGM respectively. Also, the standard deviations are 0.1631, 0.16378, 0.1459 and 0.1638 for N\_Geometric, N\_Stokes, N\_SCM and N GGM respectively. These values are slightly lower than those from EGM2008, particularly under N\_SCM, but the low variability indicates the best among the models.

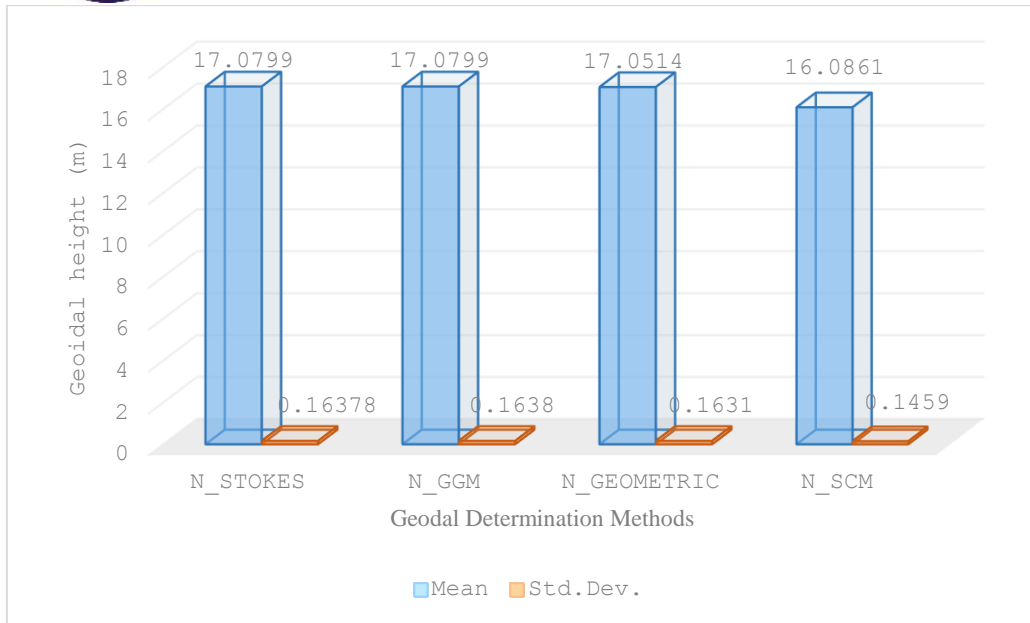


Figure 2 Geoidal Undulation Statistics Based on SGG-UGM-2

Figure 3 presents the Orthometric heights were based on SGG\_UGM\_2, the mean values are 527.5496, 527.5212 and 528.5149 for H\_Spirit Levelling, H\_STOKES and H\_SCM respectively. Also, the Standard deviations are 2.2586, 2.2621 and 2.2609 for H\_Spirit Levelling, H\_STOKES and H\_SCM respectively. However, H\_Spirit Levelling with standard deviation of 2.2586. The Standard deviations are closer to each other around 2.26m across the methods.

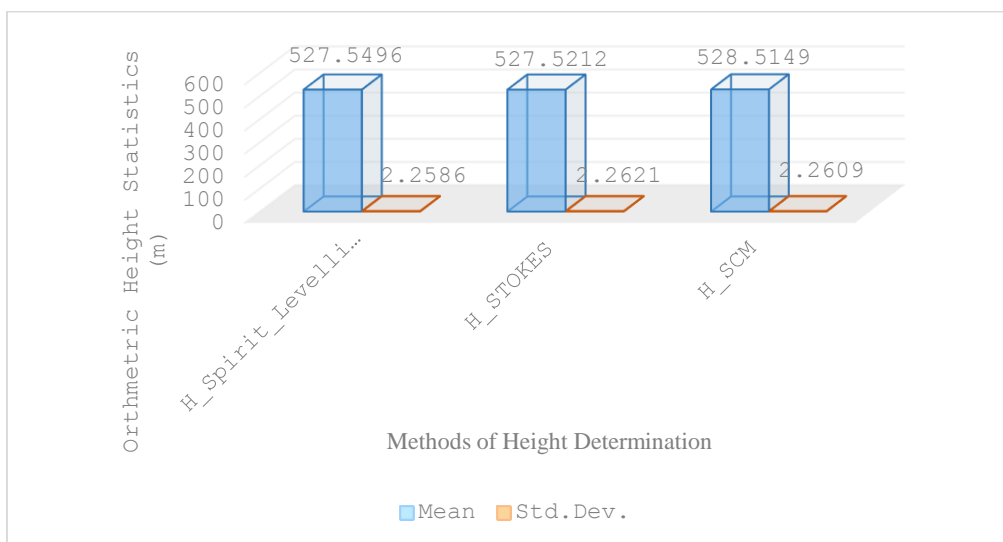


Figure 3. Orthometric Height Statistics based on SGG-UGM-2

Figure 4, presents the XGM2019e\_2156 model based on geoidal undulation graphically, the mean values are 17.0514, 16.9536, 16.9536 and 15.9600 for N\_Geometric, N\_Stokes, N\_GGM, and N\_SCM respectively. Also, the Standard deviations are 0.1631, 0.1626, 0.1626 and 0.1457 for N\_Geometric, N\_Stokes, N\_GGM, and N\_SCM respectively. However, the N\_SCM with Standard deviation of 0.1457 is more clustered to the mean, reflecting precision despite the underestimation trend.



Figure 4. Geoidal Undulation Statistics Based on XGM2019e\_2156

Figure 5, the Orthometric Heights of XGM2019e\_2156 shows that, the mean values are 528.6411, 527.6475 and 527.5496 for H\_SCM, H\_Stokes and H\_Spirit Levelling respectively. Also, the Standard deviations are 2.2641, 2.2653 and 2.2586 for H\_SCM, H\_Stokes and H\_Spirit Levelling respectively. The Standard deviations are closer to each other at about 2.26m, but the mean shift highlights less agreement with H\_Spirit levelling observations.

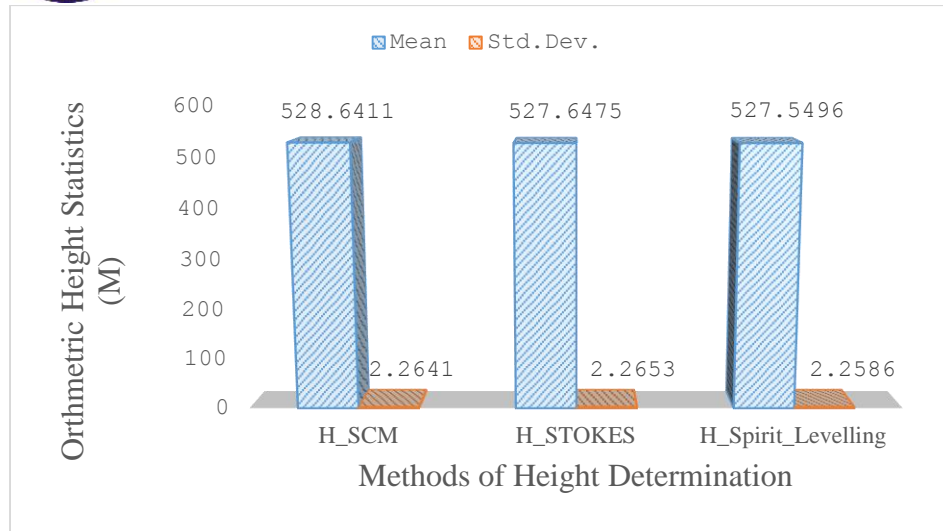


Figure 5. Orthometric Height Statistics Based on XGM2019e\_2156

### 3.2 Discussion

The results indicate that geoid undulations derived using the SCM consistently exhibited the lowest standard deviations across all GGMs, with values ranging between 0.1457 m and 0.1481 m. This demonstrates superior clustering and precision compared to both the geometric and Stokes based solutions. Similar observations have been reported by Janecka, (2023) and Aleem, (2022), who noted that SCM effectively balances long- and short-wavelength gravity signals. Under the EGM2008 model, the SCM showed the most stable geoid estimates, aligning with findings by Lestari *et al.*, (2023) that highlighted the robustness of EGM2008 in regional applications. The SGG-UGM-2 model produced slightly lower standard deviations than EGM2008, suggesting improved local performance, consistent with recent evaluations of combined satellite terrestrial models.

Orthometric height comparisons revealed that Stokes-derived heights had mean values closest to spirit levelling, particularly under EGM2008 and SGG-UGM-2. The similarity in standard deviations ( $\approx 2.26$  m) across all methods indicates uniform precision, though systematic mean shifts were observed for XGM2019e\_2156. While XGM2019e\_2156 benefits from higher global resolution (Yoldoshaliyev, 2023), the observed discrepancies suggest the



need for regional refinement when applied to local height systems. The findings of this study confirms that combined gravimetric approaches outperform purely geometric solutions, especially when SCM is employed with appropriate GGMs.

## **4.0 CONCLUSION AND RECOMMENDATIONS**

### **4.1 Conclusion**

This study evaluated geometric and gravimetric approaches for geoid and orthometric height determination using EGM2008, SGG-UGM-2, and XGM2019e\_2156 models. The results demonstrate that the Spectral Combination Method consistently provides the most precise geoid estimates, as evidenced by the lowest standard deviations across all models. Orthometric heights derived from the Stokes approach showed the closest agreement with spirit levelling observations, particularly for EGM2008 and SGG-UGM-2. Although XGM2019e\_2156 offers enhanced global resolution, regional discrepancies highlight the importance of local validation. The study confirms that integrating geometric and gravimetric techniques significantly improves height determination accuracy and supports their application in geodetic and engineering practices.

### **4.2 Recommendations**

Based on the comparative analysis of geometric and gravimetric approaches for geoid and orthometric height modelling in Hong Local Government Area, the following recommendations are made:

- i. The consistent performance of the Simple Combination Method (SCM), which produced the lowest standard deviations across all evaluated Global Geopotential Models (GGMs), indicates that combined gravimetric techniques should be preferred for precise local geoid determination. In practical applications, SCM integrated with regionally stable GGMs such as EGM2008 and SGG-UGM-2 is



recommended for height determination in the study area due to their closer agreement with spirit levelling observations.

- ii. Although the XGM2019e\_2156 model offers improved global resolution, the observed systematic mean shifts suggest that its direct application for local height determination should be preceded by regional bias correction using GPS/levelling control points.
- iii. Furthermore, the demonstrated reliability of gravimetric-derived orthometric heights supports a gradual reduction in dependence on conventional spirit levelling, particularly for large-area surveys and geodetic projects. The methodology adopted in this study is therefore recommended as a viable framework for supporting the modernization of regional and national vertical reference systems in Nigeria.



### **4.3 Future Research Directions**

Future studies should extend the present analysis to a wider regional scale by:

- i.** Incorporating additional Local Government Areas within Adamawa State and adjoining regions. Such expansion would allow assessment of spatial continuity and robustness of local geoid models.
- ii.** The integration of denser terrestrial gravity data is also recommended to improve the recovery of short wavelength gravity signals, especially in areas of complex topography.
- iii.** Further research should evaluate the performance of emerging high resolution and time variable global geopotential models for local geoid and orthometric height determination. In addition, the influence of refined terrain corrections, subsurface density modelling, and uncertainty propagation on geoid accuracy warrants detailed investigation.
- iv.** Finally, future work should explore the linkage of locally derived geoid models to the International Height Reference Frame to enhance global compatibility and support standardized height system realization.



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